Generalized, Basis-Independent Kinematic Surface Fitting

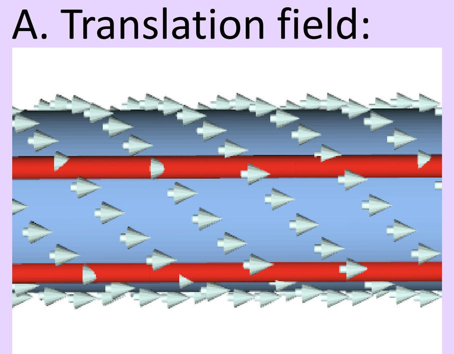
James Andrews, Carlo Séquin

Background

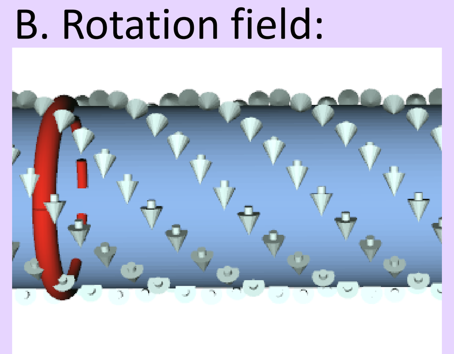
A **kinematic surface** is tangent everywhere to some easily parameterizeable, linear velocity field over space

Example: A cylinder is tangent everywhere to:

A. Translation field:

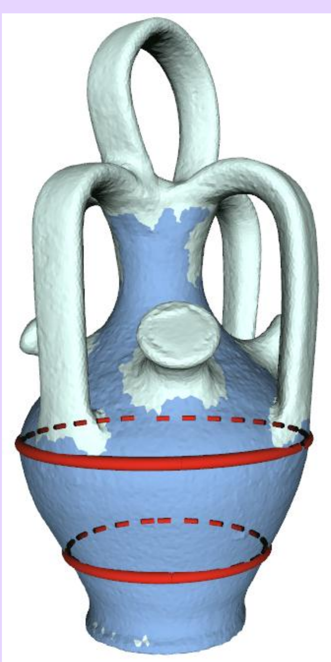


B. Rotation field:

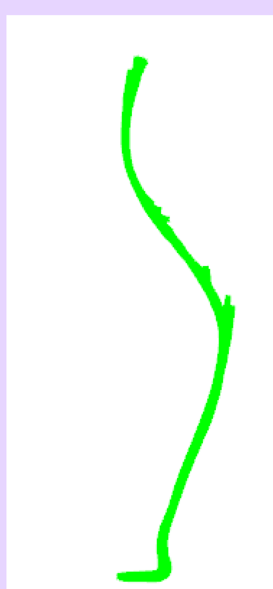


Kinematic surface fitting entails:

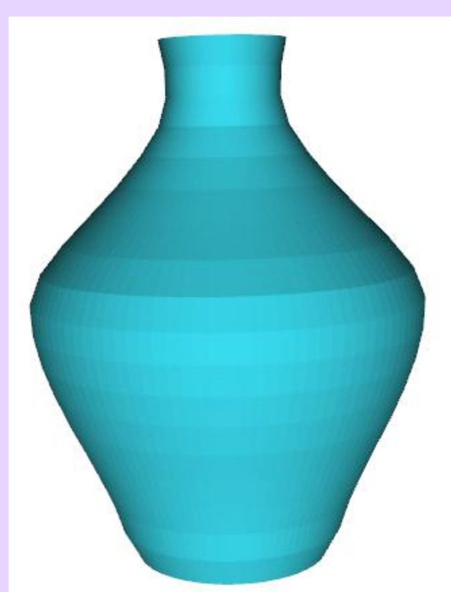
(A) Find a kinematic motion field (red streamlines)



(B) Project data to common plane; fit 'generator curve'

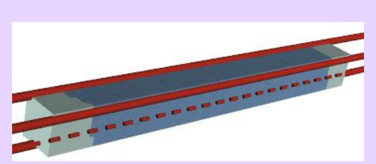


(C) Advect generator curve along motion field; create kinematic surface



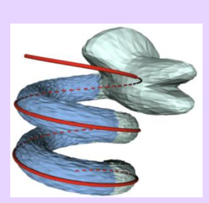
Kinematic surface field types:

Constant field



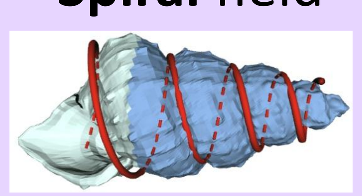
$\mathbf{v}(\mathbf{p}) = \mathbf{c}$

Helical field



$\mathbf{v}(\mathbf{p}) = \mathbf{r} \times \mathbf{p} + \mathbf{c}$

Spiral field



$\mathbf{v}(\mathbf{p}) = \mathbf{r} \times \mathbf{p} + \mathbf{c} + \gamma \mathbf{p}$

[H. Pottmann, J. Wallner, **Computational Line Geometry**, Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2001.]

ACKNOWLEDGEMENTS

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Previous Methods

Common kinematic motion fitting method:

Parameterize $\mathbf{v}(\mathbf{p})$ by vector $\mathbf{m}$.

Solve: $\underset{\mathbf{m}}{\operatorname{argmin}} \sum_i (\mathbf{v}(\mathbf{p}_i) \cdot \mathbf{n}_i)^2$

subject to some quadratic constraint, q(**m**)=**1**

Solve as small generalized eigenvalue problem.

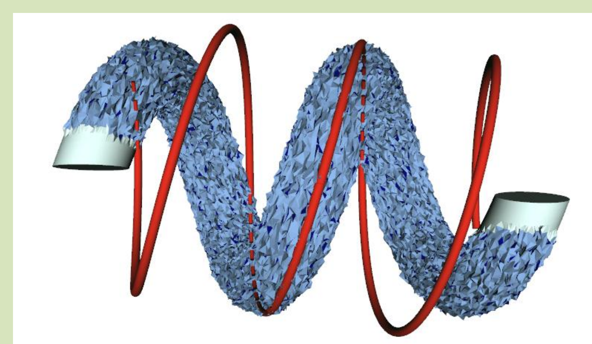
Results depend on quadratic constraint.

Biased by scale of $\mathbf{v}(\mathbf{p})$. With noise, biases cause erroneous fits.

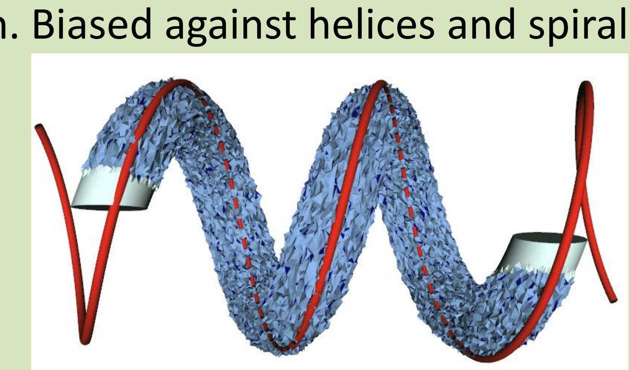
Rotation constraint: $\| \mathbf{r} \|^2 = 1$ (Rotation axis has magnitude 1.)

[Pottmann and Randrup, 98]

Works for surfaces of revolution. Biased against helices and spirals:



Rotation Constraint



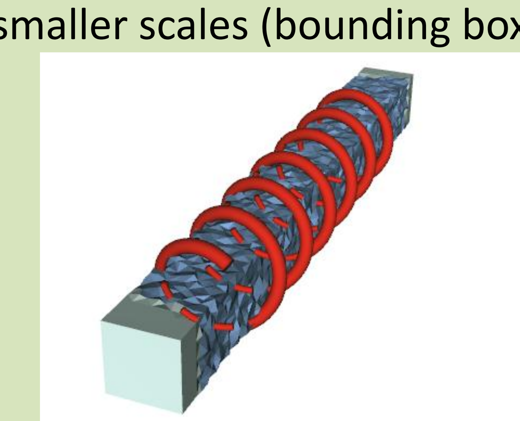
Taubin Constraint

Unit constraint: $\| \mathbf{m} \|^2 = 1$ (Complete param. vec. has magnitude 1.)

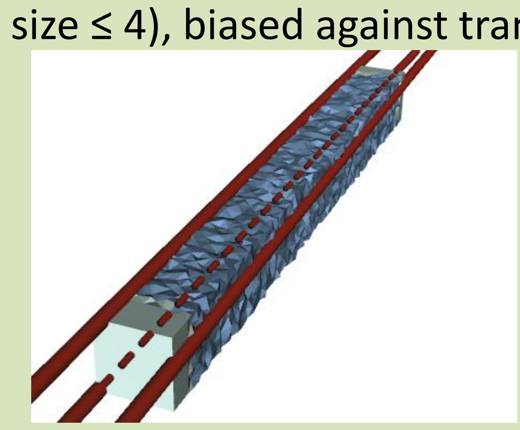
[Gelfand and Guibas, 04; Hofer et al. 05]

Works for many cases. Biases depend on scale:

At smaller scales (bounding box size ≤ 4), biased against translation:



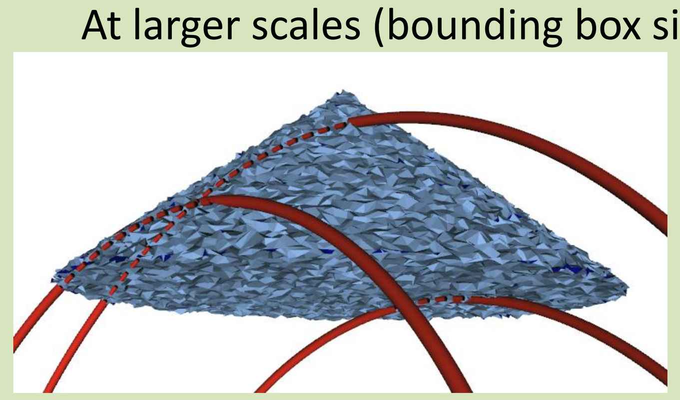
Unit Constraint



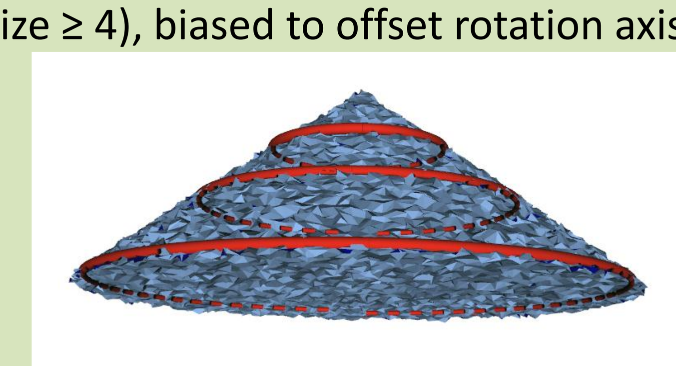
Taubin Constraint

(cause: rotation/scaling has smaller vel. at pts near axis, so smaller error)

At larger scales (bounding box size ≥ 4), biased to offset rotation axis:



Unit Constraint



Taubin Constraint

(cause: rotation axis magnitude is smaller to permit offset)

Rescaling not the answer: No fixed scale for unit constraint works for all examples!

Improved Methods

We adapt methods previously used for algebraic surface fitting and other computer vision problems:

1. An Improved Quadratic Constraint

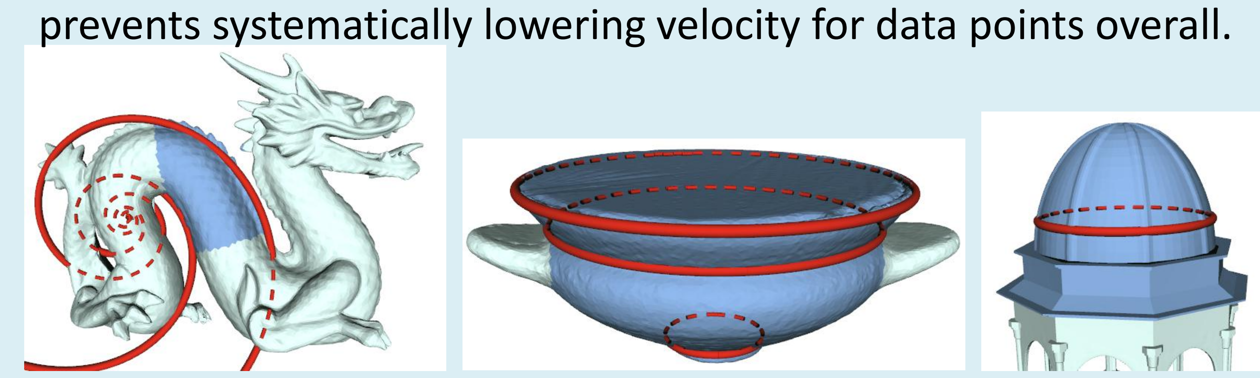
Taubin Constraint: $\sum_i \| \nabla_{\langle \text{w.r.t. data params} \rangle} (\text{error}) \|^2 = 1$

[Taubin, 91]

For KSF problem: $\sum_i \| \mathbf{v}(\mathbf{p}_i) \|^2 = 1$ (error in normal only)

"Improved" because:

Basis independent & less bias: Constraining avg. velocity prevents systematically lowering velocity for data points overall.



More examples fit with Taubin constraint

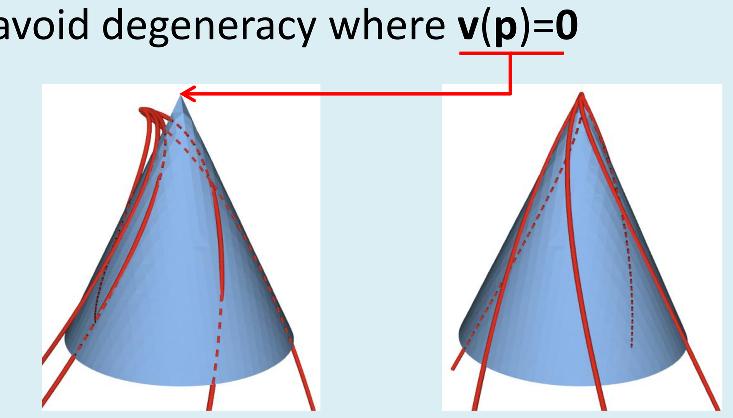
2. An Iterative Method:

HEIV Method: Iteratively solve w/ Taubin's method

Reweight error for each data point to compensate for excess weight, based on last iteration's solution.

[Leedan and Meer, '00]

Detail: Use error in position, not just normal, to avoid degeneracy where **v**(**p**)=**0**



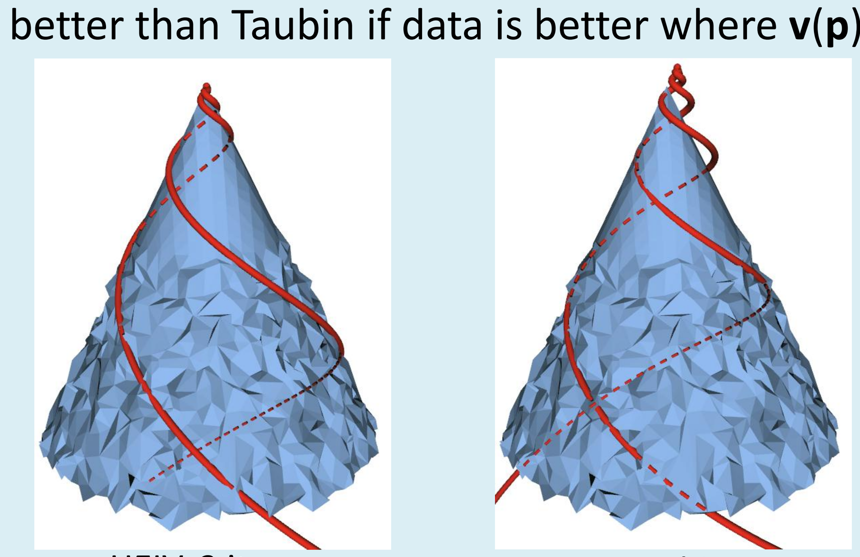
Normal error only

Normal+position error

Constraint becomes: $\sum_i w_{\mathbf{p}} \| \nabla_{\mathbf{p}} (\mathbf{v}(\mathbf{p}_i) \cdot \mathbf{n}_i) \|^2 + \| \mathbf{v}(\mathbf{p}_i) \|^2 = 1$

A small weight for pos'n error; we use: $w_{\mathbf{p}} := .001$

HEIV fits better than Taubin if data is better where **v**(**p**) is smaller:



HEIV, 3 iters

Taubin

Generalization

New velocity fields can be fit:

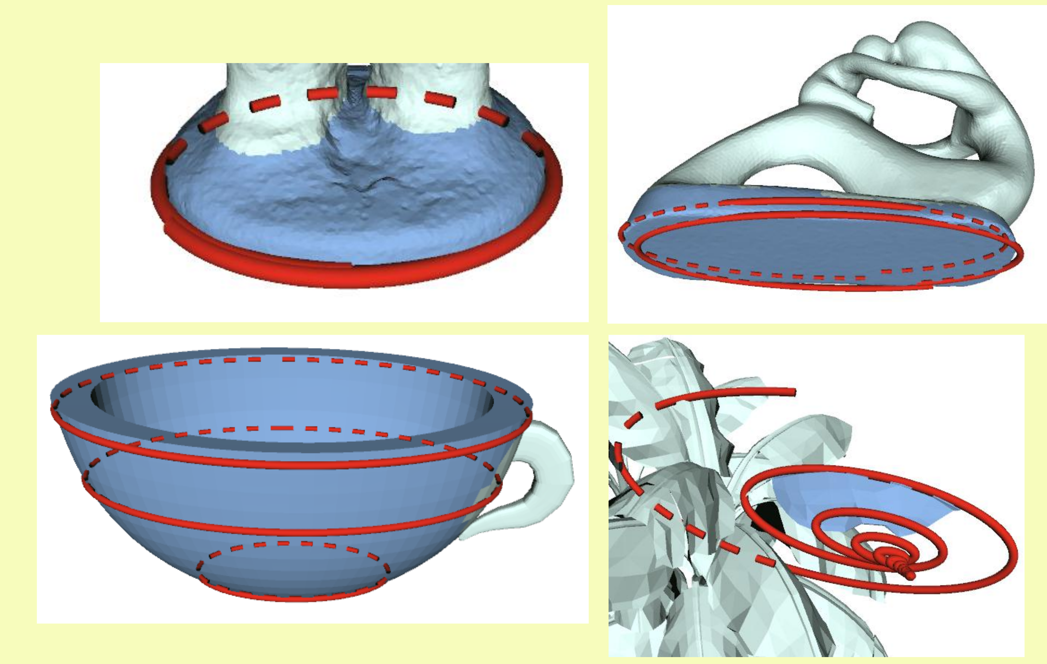
For example, an elliptical surface of revolution would be permitted by scaling a helical field:

$$\mathbf{v}(\mathbf{p}) := \mathbf{S}^{-1}(\mathbf{r} \times (\mathbf{S}\mathbf{p})) + \mathbf{c}$$

A general, linear form of this is:

$$\mathbf{v}(\mathbf{p}) := \mathbf{A}\mathbf{p} + \mathbf{c}$$

The Taubin constraint works on this new field:



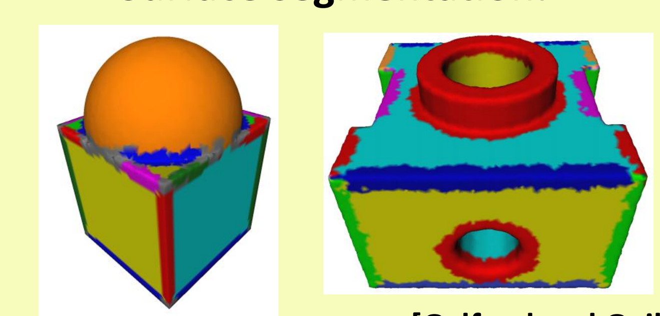
Elliptical revolutions fit using Taubin constraint

Applications

These improvements can robust-ify previous kinematic surface fitting applications.

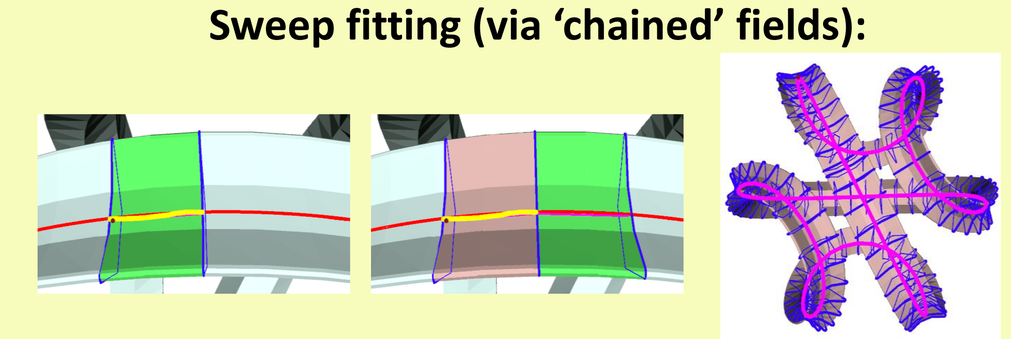
Such applications include:

Surface segmentation:



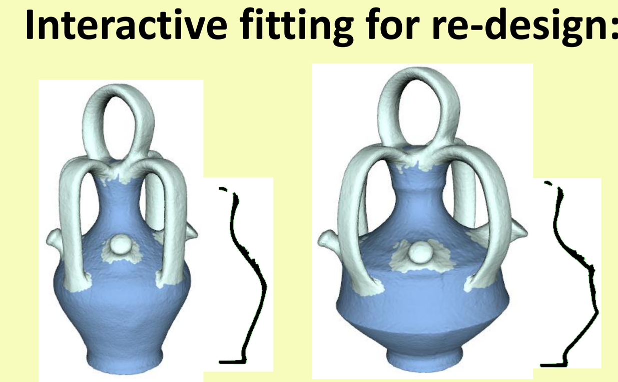
[Gelfand and Guibas, 04]

Sweep fitting (via 'chained' fields):



e.g. **[Pottmann, Chen, and Lee, 98]**

Interactive fitting for re-design:



e.g. **[Andrews, Jin, Sequin, 12]**