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Type-Constrained Direct Fitting of Quadric Surfaces

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Introduction

Efficient fitting of quadric surfaces to unstructured point clouds or triangle meshes is an important component of many reverse engineering systems [6]. Users may prefer a given surface be fit by a specific quadric type: for example, they may want to ensure the quadric is a cone, ellipsoid, or a rotationally-symmetric subtype (spheroid, circular cone, etc). Methods for type-specific quadric fitting are scattered throughout the literature: some papers handle spheres, circular cones and cylinders [5]; a few others handle ellipsoids [7] or hyperboloids [1]; non-circular cones and general rotationally symmetric quadrics are not typically discussed. In this paper, we present a thorough catalog of type-specific quadric fitting methods, including new methods that handle neglected quadric types, and improvements to previously proposed methods for ellipsoid- and hyperboloid-specific fitting methods.

Main Idea

Because fitting quadric surfaces is a non-linear problem, efficient quadric-fitting methods typically work in two steps: (1) a linear, direct method (typically an "algebraic" fitting method) generates an initial guess and then (2) a non-linear, iterative method is used to refine that guess [3,5]. Chernov and Ma presented efficient, non-linear optimization techniques to handle the non-linear optimization step for any quadric type [3]. The remaining challenge is the first step: generating an 'initial guess' that matches the desired quadric type, and is as close as possible to the error-minimizing result. We present direct fitting methods to quickly generate accurate initial guesses for a complete catalog of quadric surface types. Our catalog includes hyperboloids, ellipsoids, elliptical/hyperbolic paraboloids, elliptical/parabolic/hyperbolic cylinders, general cones, planes, and the rotationally-symmetric quadric sub-types (such as spheres, spheroids and circular cones).

All methods in our catalog use the basic framework of Taubin's method for algebraic surface fitting: Given some implicit formula for a quadric $f(\mathbf{p}) = 0$, solve a small generalized eigenvalue problem to minimize the squared algebraic distance $\sum f(\mathbf{p}_i)^2$ under the normalization $\sum || \nabla f(\mathbf{p}_i) ||^2 = 1$ [8]. Taubin's method has very low fitting bias, and has been established as one of the most effective direct fitting methods in practice [2,3].

Our fitting methods handle a wide range of quadric types with just two high-level strategies. In the first part of our paper, we show how quadratic constraints and a closed-form line search in parameter space allow us to effectively fit hyperboloids, ellipsoids, and paraboloids. In the second part, we show that transforming the problems to more convenient spaces and reducing the parameters used in fitting allows us to handle all remaining cases.

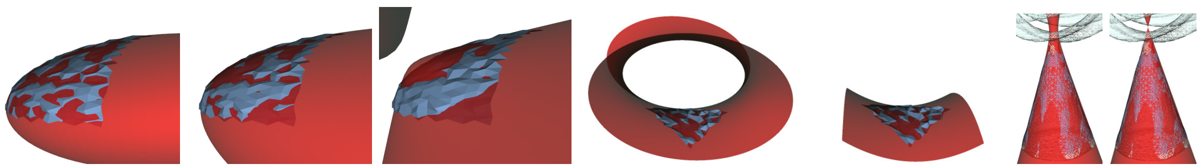
Fitting Method for Hyperboloids, Ellipsoids, and Paraboloids

Previous direct fitting methods for ellipsoids and hyperboloids have replaced Taubin's normalization with type-specific normalizations that ensure the result has the desired quadric type [1,7]. These methods guarantee a hyperboloid or ellipsoid, but the type-specific normalizations introduce more bias than Taubin's method, leading to poor results for data with even small noise (Fig. 1c). We improve upon these methods by taking the best results of previous approaches and using a direct method (derived from [4]) to search a linear subspace of those results for a better result. We also show that, when Taubin's method does not return a quadric of the desired type, the best quadric of that type (under Taubin's metric) must be on the type's *boundary*: e.g., if we want to fit a hyperboloid and Taubin's method returns an ellipsoid, then the best hyperboloid is within ϵ of a paraboloid (Fig. 1b). Therefore, we show that ellipsoid- and hyperboloid-specific fitting is equivalent to paraboloid-specific fitting. We fit all three quadric types with the same framework.

Fitting Methods for Lower-Dimensional Quadric Types

The remaining quadric types all exist in lower-dimensional sub-spaces of the full 9-dimensional space of quadrics [6]. The key to efficiently fitting these quadric types is to express the quadric with fewer parameters, such that only quadrics of the specified type can be generated, and then apply the standard algebraic fitting procedures on that parametrically-reduced form. For example, for planes and spheres we can simply drop and combine terms from the standard implicit quadric function to arrive at a plane- or sphere-specific function (e.g. $f(x,y,z) = a(x^2 + y^2 + z^2) + bx + cy + dz + e$ for spheres).

For most other lower-dimensional quadrics, the required low-dimensional space is more complicated: for example, there is no known linear-least squares method to fit circular cones and cylinders to a point cloud using just point positions [6]. However, there are linear-least squares methods for fitting such shapes to point clouds with normals [2,5]. For dense point clouds and polygonal meshes we can estimate normals (e.g. by local plane fitting, or averaging triangle normals), and then use a two-step process to fit the quadric. First, we estimate key parameters of the quadric using a direct "kinematic surface fitting" procedure that can determine properties such as a rotation symmetry axis (for a rotationally symmetric quadric), the direction in which the shape does not change (a cylinder axis) or the central point of scaling (for a general cone) [2]. Second, we transform the data to a more convenient space, and we perform the standard algebraic fit in that space. In the transformed space, it is possible to reduce the quadric parameters as we did for planes and spheres. For example, to fit general cones we translate the data so that the cone apex is at the origin, and then fit a quadric with the linear and constant parameter terms dropped: $f(x,y,z) = ax^2 + by^2 + cz^2 + dxy + exz + fyz$ (Fig. 1g).



(a) (b) (c) (d) (e) (f) (g)

Fig 1: Examples of quadric fitting (result in red) applied to a noisy data (blue mesh). (a) General quadric fitting using Taubin's method fits an ellipsoid; we use this as our ellipsoid-specific fit. (b) Our hyperboloid-specific fit; it is also within ϵ of our best paraboloid-specific fit. (c) The hyperboloid-specific fit of Allaire et al. [1] has significantly higher error. (d) General quadric fitting applied to a noisy hyperbolic paraboloid results in a hyperboloid. (e) Our direct hyperbolic paraboloid fit finds an exact hyperbolic paraboloid. (f) General quadric fitting applied to a noisy cone results in a hyperboloid with a non-zero neck at the top. (h) Our cone-specific fitting produces an exact cone.

Conclusions

We provide a practical guide for type-specific quadric fitting. We show that two straightforward, high-level approaches are general enough to fit a large range of quadric types. Our methods for ellipsoids and hyperboloids improve the state of the art, while our methods for paraboloids, general cones and rotationally symmetric quadrics fill gaps in the literature. We show results that demonstrate our methods give reasonable fits in practice.

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